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JX-003-1015003 Seat No. _____

Third Year B. Sc. (Sem. V) (CBCS) Examination

October - 2019

Mathematics : BSMT - 07 (A)

(Boolean Algebra & Complex Analysis - I)

(New Course)

Faculty Code : 003

Subject Code : 1015003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Numbers written to the right indicate full marks of the question.

1 (A) Answer the following short answer questions : **4**

(1) Define : Reflexive Relation.
(2) Define : Irreflexive relation.
(3) Define : Poset
(4) Find the greatest and least elements of the poset $\left(\{1,2,3,4,5,6\},/\right)$

(B) Answer in Brief : (Any **One**) **2**

(1) If $\left(L_1,*,\oplus,0,1\right)$ is a bounded lattice then show that $a*1=a$ and $a\oplus 1=1$
(2) Define Complemented lattice and give an example.

(C) Answer in Detail : (Any **One**) **3**

(1) Show that $\left(S_{30},D\right)$ is a lattice.
(2) State and Prove isotonicity property.

(D) Answer in Brief : (Any **One**) **5**

(1) Let $\left(L,*,\oplus\right)$ be a lattice. For any two elements $a,b\in L, glb\left\{a,b\right\}=a*b$ and $\text{lub}\left\{a,b\right\}=a\oplus b$ with respect to the partial ordering R on L.
(2) Prove that product of two lattices is also a lattice.

2 (A) Answer the following short answer questions : **4**

(1) Define : Boolean Algebra

(2) State D'morgan's Law for Boolean Algebra.

(3) Define : Atom in Boolean Algebra.

(4) If a and b are two distinct atoms then what is the value of $a * b$?

(B) Answer in Brief : (Any **One**) **2**

(1) For $\forall a, b, c \in B$ Prove : (i) $a * (a' \oplus b) = a * b$

(ii) $(a * b) \oplus (a * b') = a$

(2) Define : Boolean homomorphism

(C) Answer in Detail : (Any **One**) **3**

(1) For a finite Boolean Algebra $(B, *, \otimes, ', 0, 1)$, If x is a nonzero element of B then show that $\exists$ an atom $a \in B$ such that $a \leq x$

(2) If $(B, *. \oplus, ', 0, 1)$ is a Boolean Algebra, then for any $x_1, x_2 \in B$ show that $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(D) Answer in Detail : (Any **One**) **5**

(1) If $(B, *, \oplus, ', 0, 1)$ is a finite Boolean Algebra with A as the set of atoms of B then prove that Boolean Algebra is isomorphic to the Boolean Algebra $(P(S), \cap, \cup, \phi, S)$

(2) Prove that sum of all minterms of n-variables $x_1, x_2, x_3, ..., x_n$ is 1.

3 (A) Answer the following short answer questions : **4**

(1) Evaluate : $\lim_{z \to \infty} \frac{2z + 3}{z + i}$

(2) Define : Analytic function

(3) Write Laplace Equation

(4) State C-R condition in polar form

(B) Answer in Brief : (Any **One**) **2**

(1) Show that $f(z) = 2x + ixy^2$ is no analytic.

(2) Check whether $y^3 - 3x^2y$ is harmonic or not.

(C) Answer in Detail : (Any **One**) **3**

(1) If $w = x^2 + ayx + by^2 + i\left(x^2 + dxy + z^2\right)$ is analytic then find the values of a, b, c and d.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

(D) Answer in Detail : (Any **One**) **5**

(1) Obtain Cauchy-Riemann conditions in Cartesian form.

(2) Prove : $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, z \neq 0$

$= 0 \qquad , z = 0$

Satisfied C-R conditions at origin however $f(z)$ is not analytic function at origin.

4 (A) Answer the following short answer questions : **4**

(1) State Cauchy Integral Formula.

(2) If $c : |z| = 1$ then $\int_c \dfrac{z}{2z-1} dz =$ _____

(3) If $c : |z| = 2$ then $\int_c \dfrac{z}{(z-1)(z-3)} dz =$ _____

(4) If L is the length of contour c then $L =$ _____

(B) Answer in Brief : (Any **One**) **2**

(1) Find: $\int_c \dfrac{z}{\left(9 - z^2\right)(z+i)} dz, |z| = 2$

(2) State Green's Theorem.

(C) Answer in Detail : (Any **One**) **3**

(1) Prove that $\left|\int_c \dfrac{dz}{z^2 - 1}\right| \leq \dfrac{\pi}{3}$ where C is the arc from $z = 2$ to $z = 2i$ of circle $|z| = 2$.

(2) Prove in usual notations : $\left|\int_a^b f(z)\, dz \leq \right|\int_a^b |f(z)| dz$

(D) Answer in Detail : **5**

If the complex function is analytic everywhere inside and on closed contour C and z_0 is any point lying inside C then show that $\int_c \frac{f(z)}{z-z_0} dz = 2\pi\, if(z_0)$

5 (A) Answer the following short answer questions : **4**

(1) State Fundamental Theorem of Algebra.

(2) State : Maximum Modulus Theorem.

(3) Define : Jordan Arc.

(4) If $C : |z| = 1$ then $\int_c \frac{e^{2z}}{z^3} dz$

(B) Answer in Brief : (Any **One**) **2**

(1) Evaluate : $\int_c \frac{z^2+3}{z^2(z-4)} dz$, where $C : |z| = 1$

(2) Evaluate : $\int_c \frac{z^2+1}{(z-1)^3} dz$, where $C : |z| = 1$

(C) Answer in Detail : (Any **One**) **3**

(1) State and Prove Cauchy inequality.

(2) State and Prove Liouville's Theorem.

(D) Answer in Detail : (Any **One**) **5**

(1) State and Prove Morera's Theorem.

(2) Prove : $f''(z_0) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-z_0)^{n+1}} dz$
